If $0 \le k < \dfrac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \dfrac{\pi}{2}$ is 0.1, then $k =$

(A)   1.471   (B)   1.414   (C)   1.277   (D)   1.120   (E)   0.436

$$\int_{k}^{\pi/2} \cos x \, dx = .1$$

$$\sin x \Big|_{k}^{\pi/2} = .1$$

$$\sin \tfrac{\pi}{2} - \sin k = .1$$

$$\sin \tfrac{\pi}{2} - .1 = \sin k$$
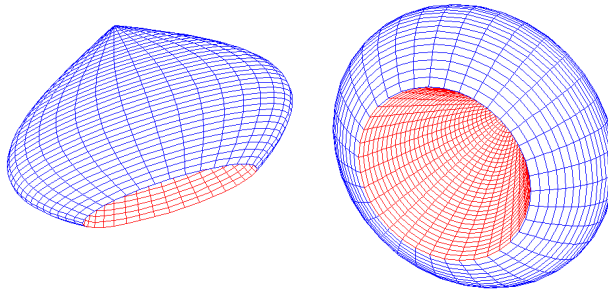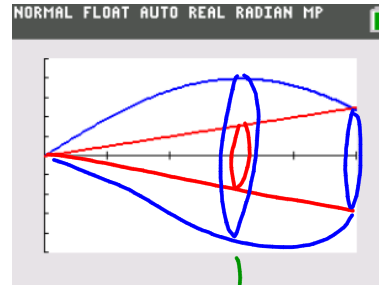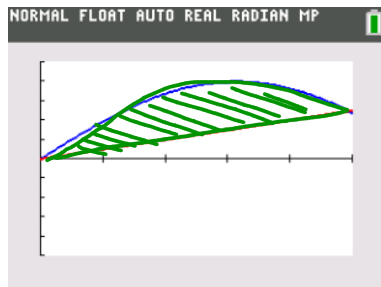
$$1 - .1 = \sin k$$

$$.9 = \sin k$$

$$k = \sin^{-1}(.9) \approx 1.120$$

7-3 Day 2 Volumes : Washers Method

Learning Targets

I find the volume of a solid that has
been rotated around an axis using
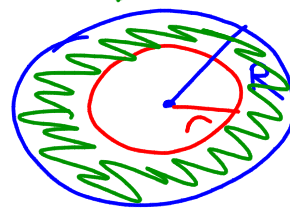the washer method.

Ex1. The region in the first quadrant enclosed by the graphs of $f(x) = 4\sin\left(\dfrac{x}{2}\right)$ and $g(x) = \dfrac{x}{2}$ is revolved around the x-axis to form a solid. Find the volume.

$A = \pi R^2 - \pi r^2$

$R = 4\sin\left(\dfrac{x}{2}\right)$

$r = \left(\dfrac{x}{2}\right)$
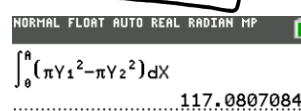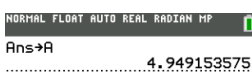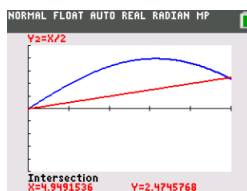
$A = \pi\left(4\sin\left(\dfrac{x}{2}\right)\right)^2 - \pi\left(\dfrac{x}{2}\right)^2$

$$\int_0^{4.9491536} \pi\left(4\sin\left(\dfrac{x}{2}\right)\right)^2 - \pi\left(\dfrac{x}{2}\right)^2 \, dx$$

$\approx \boxed{117.081} \; u^3$

NORMAL FLOAT AUTO REAL RADIAN MP
Y2=X/2
Intersection
X=4.9491536   Y=2.4745768

NORMAL FLOAT AUTO REAL RADIAN MP
Ans→A
                4.949153575

NORMAL FLOAT AUTO REAL RADIAN MP
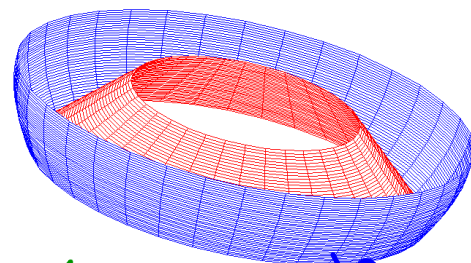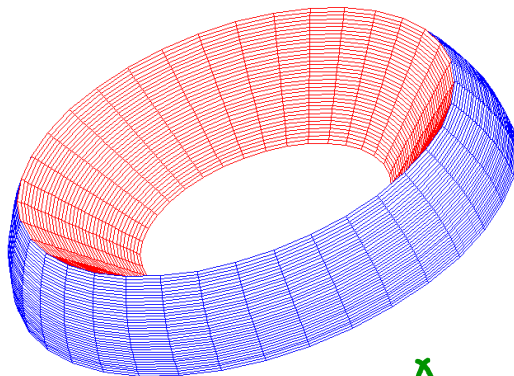$\int_0^A (\pi Y_1^2 - \pi Y_2^2) dX$
                117.0807084

Ex2. The region in the first quadrant enclosed by the y-axis and the graphs of y = cos x and y = x is revolved around the line y = -1 to form a solid. Find the volume.
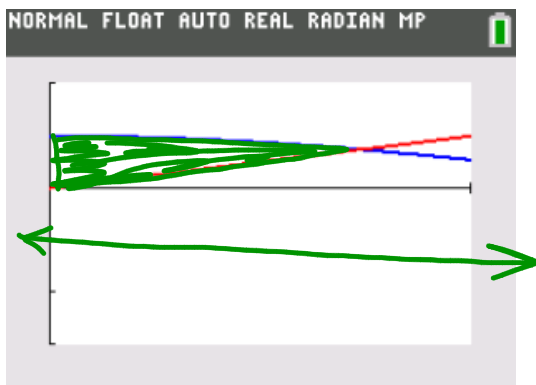
R = cos x + 1
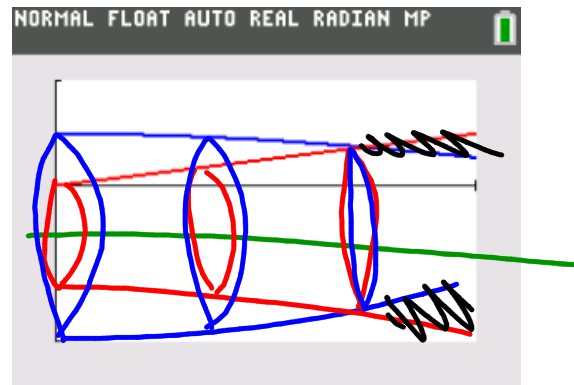
r = x + 1

$$A = \pi \left((\cos x + 1)^2 - (x+1)^2 \right)$$

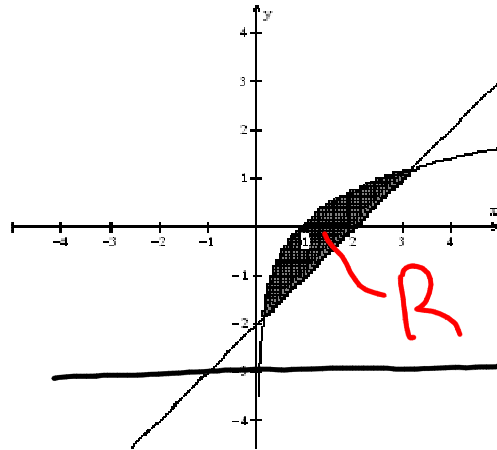$$V = \pi \int_0^{.73908513} (\cos x + 1)^2 - (x+1)^2 \, dx$$

$$\approx 1.285\pi$$

$$\approx \boxed{4.036}$$

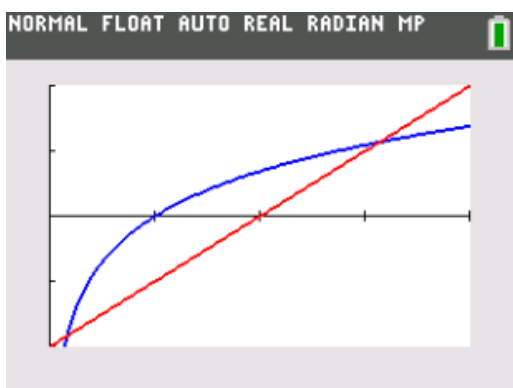Ex3. Let R be the shaded region bounded by the graph of y = ln x and the line y = x − 2 as shown.
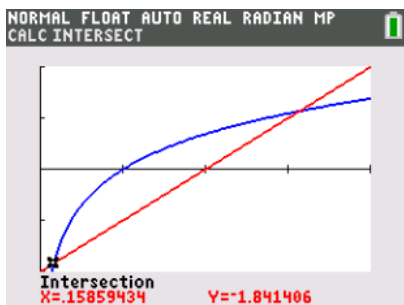
a.) Find the area of R.

b.) Find the volume of the solid when R is rotated around the horizontal line y = -3.

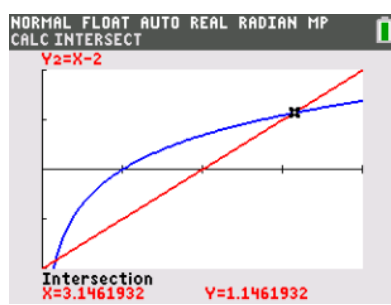c.) Find the volume of the solid when R is rotated around the y-axis.

a.)

NORMAL FLOAT AUTO REAL RADIAN MP

$$\int_A^B \ln x - (x-2)\, dx$$

$$\approx 1.949$$

NORMAL FLOAT AUTO REAL RADIAN MP
CALC INTERSECT

Intersection
X=.15859434   Y=-1.841406

NORMAL FLOAT AUTO REAL RADIAN MP

Ans→A
                    .1585943396
■

NORMAL FLOAT AUTO REAL RADIAN MP
CALC INTERSECT
Y₂=X-2

Intersection
X=3.1461932   Y=1.1461932

NORMAL FLOAT AUTO REAL RADIAN MP

Ans→B
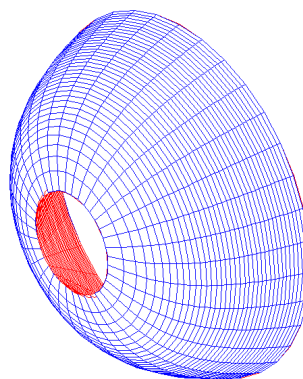   3.146193221
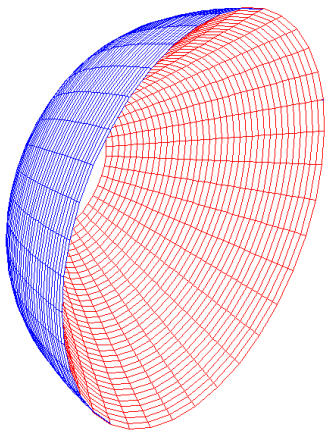■

b.)

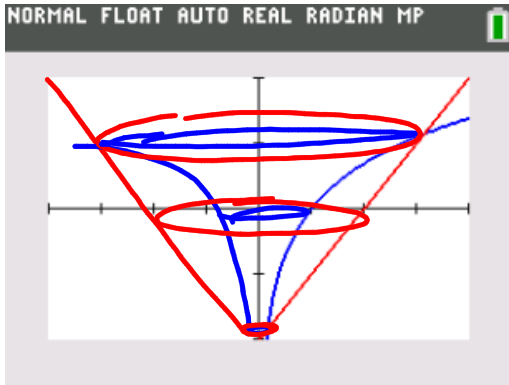NORMAL FLOAT AUTO REAL RADIAN MP
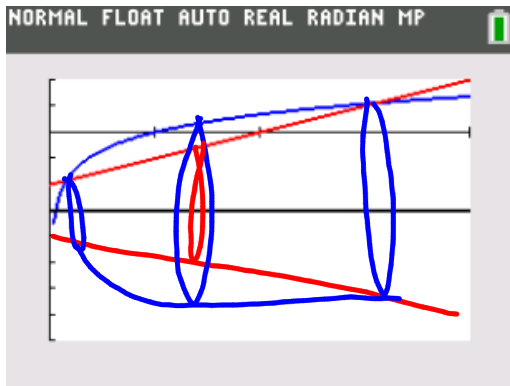
$R = \ln x + 3$

$r = (x - 2) + 3$

$r = x + 1$

$$= \pi \int_A^B (\ln x + 3)^2 - (x + 1)^2 \, dx$$

$$\approx 34.178$$

c.)
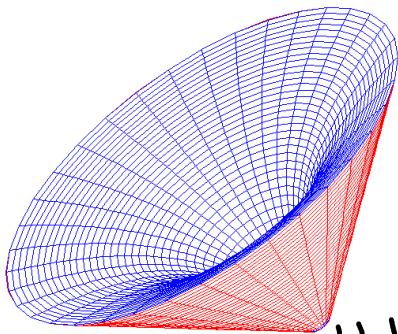

$y = \ln x$
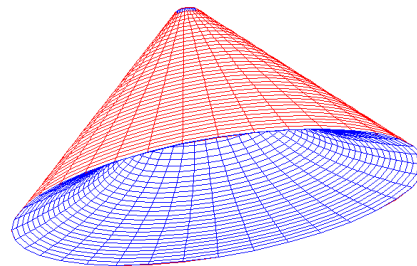
$x = e^y$

$y = x - 2$

$x = y + 2$

1.14619

$$\pi \int_{-1.84406}^{1.14619} (y+2)^2 - (e^y)^2 \, dy$$

$$\approx \boxed{17.099}$$

Homework

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